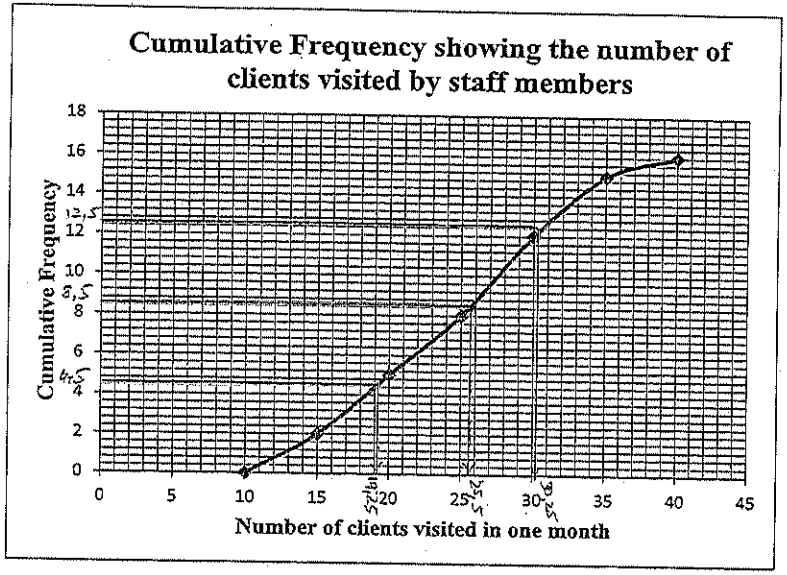
QUESTION 1

1.1 Ros is the sales manager for a print and design company. His staff members must report about the number of clients they visit every month. Below is the ogive representing the collected data in one month.



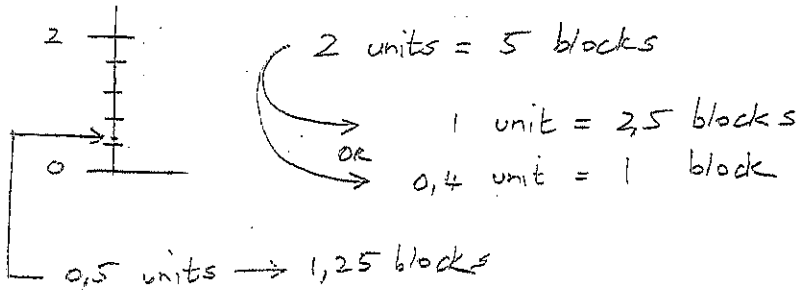
1.1 How many staff members were there? (1)

1.2 Determine the semi-interquartile range. (3)

1.3 Draw a box whisker diagram for the data. (3)

1.4 Comment on the distribution of data using your result in QUESTION 1.3. (1)
[8]

NOTE: *



Solns

1.1. 16 ✓ staff members 1

1.2. $T_1; \dots; T_{16}$ $M = T_{\frac{1}{2}(1+16)}$
$= T_{8,5}$

$T_1; \dots; T_8$ $T_9; \dots; T_{16}$

$Q_1 = T_{\frac{1}{2}(1+9)}$ $Q_3 = T_{\frac{1}{2}(9+16)}$

$= T_{4,5}$ $= T_{12,5}$

$= 19,25$ ✓ $= 30,25$ ✓

$\therefore$ Semi IQR $= \frac{IQR}{2}$ $\frac{Q_3 - Q_1}{2}$

$= \frac{30,25 - 19,25}{2}$

$= 5,5$ ✓ 3

$\approx$ 6 clients $\in \mathbb{N}$

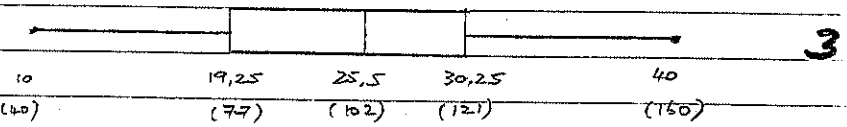
1.3. min assume 10 ✓ shape

Q_1 19,25

M 25,5 ✓ ✓ scale

Q_3 30,25

max assume 40 (Scale x 4 → mm)



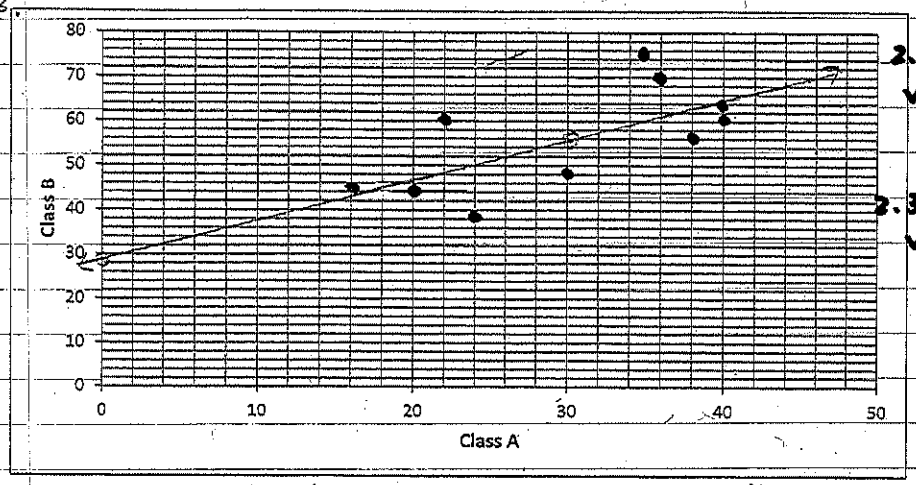
3

1.4. Skewed to the left ✓ 1

QUESTIONS 2.1 AND 2.3

2.1.

2.3



2.1. ✓✓ 2

2.3. ✓ 1

2.2. $A = 29,22$ ✓both $B = 0,89$ $\therefore$ ✓eqn $y = 29,22 + 0,89x$ 2

2.3. $\bar{x} = 30,1$ $\bar{y} = 55,9$ $\therefore$ $(30,1 ; 55,9)$

other point $x = 0$ $y = 29,22$ $\therefore$ $(0 ; 29,22)$

- see graph

2.4. $r = 0,66$ ✓✓ 2

2.5. Class B : ✓✓ $\bar{y} = 55,9$ ✓✓ $\sigma_y = 11,36$ 4

2.6. Class B : 38; 44; 45; 48; 56; 60; 60; 63; 70; 75 (2)

$\bar{y} - 1\sigma_y = 55,9 - 11,36$ $\bar{y} + 1\sigma_y = 55,9 - 11,36$

$= 44,54$ ✓ $= 67,26$

$\therefore$ within are 45; 48; 56; 60; 60; 63

$\therefore$ 6 scores ✓ 2

QUESTION/*VRAAG* 3

3.1	$y = 9$	✓ answer/*antwoord*	(1)	1
3.2	AB = 11 units/*eenhede* **OR/*OF*** $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AB = \sqrt{(11 - 0)^2 + (9 - 9)^2}$ AB = 11 units/*eenhede*	✓ answer/*antwoord*	(1)	1
3.3	D(13; 9)	✓ answer/*antwoord*	(1)	1
3.4	Area of ΔABC = ½ AB.CD (*Oppervlakte van . . .*) = ½(11 × 8) = 44 square units/*vierkante eenhede* CD = 8	✓ length of CD / *lengte van CD* ✓Substitute into correct formula / *Vervanging in korrekte formule* ✓answer/*antwoord*	(3)	3
3.5	$x_M = \frac{0+13}{2} = 6,5$ $y_M = \frac{9+1}{2} = 5$ A(0; 9) C(13; 1) M(6,5; 5)	✓substitution into midpoint formula/ *vervanging in middelpunt formule* ✓answer/*antwoord*	(2)	2
3.6	$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-1}{0-13}$ A(0; 9) C(13; 1) $m_{AC} = -\frac{8}{13}$ ∴ $m_\perp = \frac{13}{8}$ $y = \frac{13}{8}x + c$ Sub M(6,5; 5) $5 = \frac{13}{8}(6,5) + c$ $-\frac{89}{16} = c$ ← −5,56 ∴ $y = \frac{13}{8}x - \frac{89}{16}$	✓ m_{AC} ✓ correct gradient of perpendicular bisector/*korrekte gradiënt van middelloodlyn* ✓ Substitution into correct formula/ *Vervanging in korrekte formule* ✓answer/*antwoord*	(4)	4
3.7	$y = \frac{13}{8}(11) - \frac{89}{16}$ $y = \frac{143}{8} - \frac{89}{16}$ $y = \frac{197}{16} \neq 9$ No, it does not pass through B/*Nee, dit gaan nie deur B nie.*	B(11; 9) $12,31 = \frac{197}{16}$ ✓substitution into correct equation / *vervanging in korrekte vergelyking* ✓ answer/*antwoord* (justification/ *regverdiging*)	(2)	2

3.8	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ B(11; 9) C(13; 1) $= \frac{9-1}{11-13}$ $= \frac{8}{-2} = -4$ $\tan\theta = -4$ neg ∠ = 75,96 … ° $\theta = 104,04°$ $A\hat{B}C = 104°$ alt ∠'s = , 11 lines	✓ m_{BC} ✓ $\tan\theta$ ✓ answer/*antwoord*	(3)	3
3.9	$y = -\frac{8}{13}x + c$ ⊥ to AC Sub D(13; 9) $9 = -\frac{8}{13}(13) + c$ ∴ $17 = c$ $y = -\frac{8}{13}x + 17$	✓ substitution/*vervanging* ✓ answer/*antwoord*	(2)	2
			[19]	

QUESTION/*VRAAG* 4

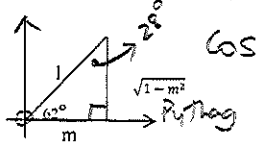
4.1	$x^2 - 2x + (-1)^2 + y^2 + 4y + (+2)^2 = a + 1 + 4$ $(x - 1)^2 + (y + 2)^2 = a + 5$ $= r^2$ $a + 5 = (5)^2$ $a = 20$	✓✓completing the square/*voltooiing van vierkant* ✓equating r^2/*gelyk stel aan* r^2 ✓$r^2 = 25$	(4)	4
4.2	M (1; -2)	✓ answer in coordinate form/ *antwoord in koördinaat vorm*	(1)	1
4.3	$x^2 - 2x + y^2 + 4y = 20$ A(x; y) $\therefore (4)^2 - 2(4) + y^2 + 4y = 20$ $16 - 8 + y^2 + 4y = 20$ $y^2 + 4y - 12 = 0$ $(y + 6)(y - 2) = 0$ ∴ y = -6 or/of y = 2 $y = 2$ $A(4; 2)$ $y > 0$	✓ substituting into correct formula/*vervanging in korrekte formule* ✓standard form/*standaardvorm* ✓ factors/*faktore* ✓correct answer/*korrekte antwoord*	(4)	4
4.4	$m_{MA} = \frac{2+2}{4-1} = \frac{4}{3}$ M(1; -2) A(4; 2) $m_{tangent} = -\frac{3}{4}$ tan ⊥ rad $y = -\frac{3}{4}x + c$ Sub A(4; 2) $2 = -\frac{3}{4}(4) + c$ $5 = c$ $\therefore y = -\frac{3}{4}x + 5$	✓$m_{MA} = \frac{4}{3}$ ✓$m_{tan.} = -\frac{3}{4}$ ✓substituting into correct formula/*vervanging in korrekte formule* ✓answer/*antwoord*	(4)	4

4.5	M(1; -2) , T(-1;-2) MT $=\sqrt{(-1-1)^2+(-2+2)^2}$ ✔ $=2$ ✔ $r=5$ $\therefore MT < r$ $\therefore$ T lies inside the circle. ✔	✓ substitution/*vervanging* ✓simplification/*vereenvoudiging* ✓ answer/*antwoord*	(3)
4.6	$(x-1)^2+(y+2)^2=25$ $(x-1+3)^2+(y+2-1)^2=25$ $(x+2)^2+(y+1)^2=25$ ✔ M(1;-2) $\xrightarrow{\begin{matrix}3\leftarrow\\1\uparrow\end{matrix}}$ (-2;-1) new centre	✓ substitution into correct formula/*vervanging in korrekte formule* ✓ ✓ simplification and answer $(x+2)$ and $(y+1)$ / *vereenvoudiging en antwoord* $(x+2)$ *en* $(y+1)$	(3)
			[19]

3

3

QUESTION/*VRAAG* 5

5.1.1	$\sin 28° = \sin(90°-62°)$ $= \cos 62°$ ✔ $= m$ ✔  Cos 62° = m = m/1	✓ equation/*vergelyking* ✓ answer/*antwoord*	(2)
5.1.2	$\cos 362° = \cos(2°)$ ✔ $= \cos(62°-60°)$ ✔ ✔ $= \cos 62° \cos 60° + \sin 62° \sin 60°$ $= \frac{1}{2}m + \frac{\sqrt{3}}{2}(\sqrt{(1-m^2)})$ ✔	✓ cos 2° ✓ 62° – 60° ✓ expansion/*uitbreiding* ✓substitution/*vervanging*	(4)
5.2.1	$\frac{\tan(360°-x).\sin(90°+x)}{\sin(-x)}$ $= \frac{-\tan x.\cos x}{-\sin x}$ ✔ ✔ $= \frac{\frac{\sin x}{\cos x}\times\cos x}{\sin x}$ ✔ $= 1$ ✔	✓ - tan x ✓ cos x ✓ - sin x ✓ $\frac{\sin x}{\cos x}$ ✓ answer/*antwoord*	(5)

4

5

5.3	$4\sin^2\theta = 3$ $\sin^2\theta = \frac{3}{4}$ $\therefore \sin\theta = \pm\frac{\sqrt{3}}{2}$ ✔ * $\therefore \theta = 120°$; $90° < \theta < 180°$ $= \cos\frac{1}{4}\theta.\sin\frac{1}{2}\theta - \tan(3\theta-45°)$ $= \cos 30°.\sin 60° - \tan(360°-45°)$ ✔ $= \cos 30°.\sin 60° + \tan 45°$ $= \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}+1$ ✔ ✔ ✔ $= \frac{3}{4}+1 = \frac{7}{4}$ 1,75	✓ simplification/*vereenvoudiging* ✓ value of/*waarde van* θ ✓substitution of/*vervanging van* θ ✓ Cos 30° $= \frac{\sqrt{3}}{2}$ ✓ Sin 60° $= \frac{\sqrt{3}}{2}$ ✓ tan 45° = 1	(6) [17]

6

QUESTION/*VRAAG* 6

	LHS $= \frac{\cos 2x}{1+\sin 2x}$ $= \frac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}$ ✔ ✔ $= \frac{\cos^2 x-\sin^2 x}{\sin^2 x+2\sin x\cos x+\cos^2 x}$ ✔ $= \frac{(\cos x+\sin x)(\cos x-\sin x)}{(\sin x+\cos x)(\sin x+\cos x)}$ ✔ $= \frac{\cos x-\sin x}{\sin x+\cos x}$	✓ expansion in numerator/ *uitbreiding van teller* ✓ expansion in denominator/ *uitbreiding van noemer* ✓$\sin^2 x+\cos^2 x$ ✓factorisation/*faktorisering*	[4]

4

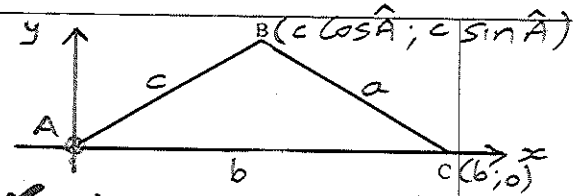
* ref $\angle = 60°$
Sin is $\pm$ in (k ∈ ℤ)
QI : X
QII : $\theta = 120° + k\,360°$
QIII : X
QIV : X
$\therefore \theta = 120°$

$90° < \theta < 180°$!

I suspect Q5 was supposed to be woc!
But it did not say so!

QUESTION/VRAAG 7

7.1



$a^2 = (c\sin A - 0)^2 + (c\cos A - b)^2$

$= c^2\sin^2 A + c^2\cos^2 A - 2bc\cos A + b^2$

$= c^2(\sin^2 A + \cos^2 A) + b^2 - 2bc\cos A$

$= c^2 \cdot 1 + b^2 - 2bc\cos A$

$= b^2 + c^2 - 2bc\cos A$

✓AD in terms of x / *AD in terme van x*
✓ c^2 (using/*gebruik van* Pythagoras)
✓ $(h^2 + x^2) = a^2$
✓substitution of x/ *vervanging van x*

(4)

7.2.1

PSQ = 90°
$Q_3 = 90° + \alpha$ ✓ Ext ∠ Δ

In ΔQSR:
$SR^2 = SQ^2 + QR^2 - 2SQ.QR.\cos SQR$
$= x^2 + x^2 - 2x.x.\cos(90° + \alpha)$ ✓ f+s
$= 2x^2 - 2x^2(-\sin\alpha)$ ✓ red^ form
$= 2x^2 + 2x^2(\sin\alpha)$
$= x^2(2 + 2\sin\alpha)$ ✓
$\therefore SR = x\sqrt{2(1 + \sin\alpha)}$

✓ angle in semi-circle/*hoek in semi-sirkel*
✓ use of cos rule/ *gebruik van cos-reël*
✓ correct substitution/ *korrekte vervanging*
✓ $x^2(2 + 2\sin\alpha)$

(4)

7.2.2

$5\sqrt{3} = x\sqrt{2(1 + \sin\alpha)}$
$5\sqrt{3} = 5\sqrt{2(1 + \sin\alpha)}$ ✓
$\sqrt{3} = \sqrt{2(1 + \sin\alpha)}$
$3 = 2(1 + \sin\alpha)$
$\frac{3}{2} = 1 + \sin\alpha$
$\frac{1}{2} = \sin\alpha$ ✓
$\alpha = 30°$ ✓

Ref ∠ = 30°
sin + in
Q I : $\alpha = 30°$
Q II : x

In ΔPSQ
$\frac{QS}{PQ} = \sin\alpha$
$PQ = \frac{QS}{\sin\alpha}$
$= \frac{5}{\sin 30°}$
$= 10$ units/*eenhede* ✓

✓Substitution/ *Vervanging*
✓ $\sin\alpha = 0,5$
✓ $\alpha = 30°$
✓ answer/*antwoord*

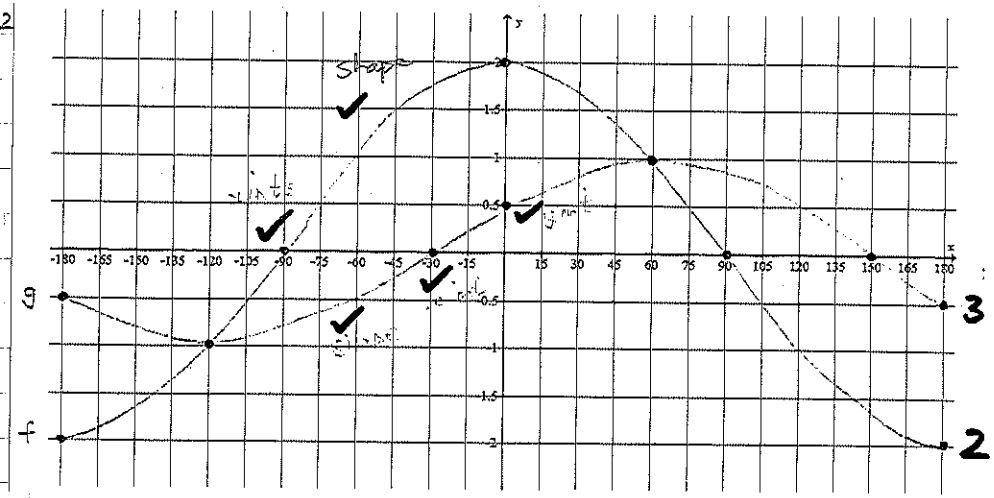
(4)

[12]

8. $f(x) = 2\cos x$ $g(x) = \sin(x + 30°)$

8.1. Amp = 2 ✓ **1**

8.2



3 **2**

8.3. $f(x) - g(x) \geq 0$

$y_f - y_g \geq 0$

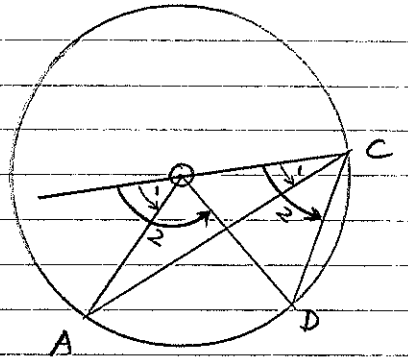
$y_f \geq y_g$ f above or = to g

$\therefore x \in [-120°; 60°]$ ✓✓ **2**

8.4. Translate f

- 2 units vertically downwards ✓
- 90° to the left (or right) ✓ **2**

9.1.



Constr: ✓ CO extended to K

$\hat{O}_1 = \hat{A} + \hat{C}_1$ ✓ Ext ∠ Δ

but $\hat{A} = \hat{C}_1$ Isos Δ, sides = radii ✓

$\therefore \hat{O}_1 = \hat{C}_1 + \hat{C}_1$

$= 2\hat{C}_1$

Similarly,

$\hat{O}_2 = 2\hat{C}_2$ ✓

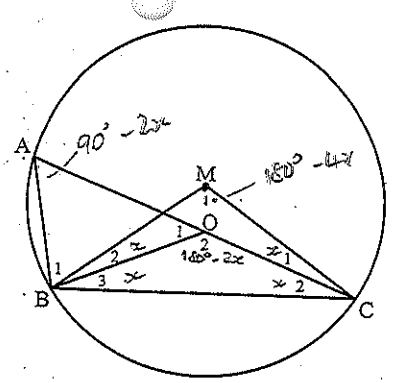
$\therefore A\hat{O}D = \hat{O}_2 - \hat{O}_1$

$= 2\hat{C}_2 - 2\hat{C}_1$

$= 2(\hat{C}_2 - \hat{C}_1)$ ✓

$= 2 \cdot A\hat{C}D$ 5

9.2



9.2.1. $\hat{B}_3 = x$ given

$\therefore \hat{C}_{1+2} = 2x$ ✓S Isos, sides = ✓R radii

$\therefore \hat{C}_1 = \hat{C}_2$ given

$= x$ ✓S

$\therefore \hat{O}_2 = 180° - 2x$ ✓SR ∠'s Δ = 180°

4

9.2.2. $\hat{M}_1 = 180° - 4x$ ✓SR ∠'s Δ = 180°

$\therefore \hat{A} = 90° - 2x$ ✓S ∠ @ ✓R centre = 2 ∠ @ ⊙'ce

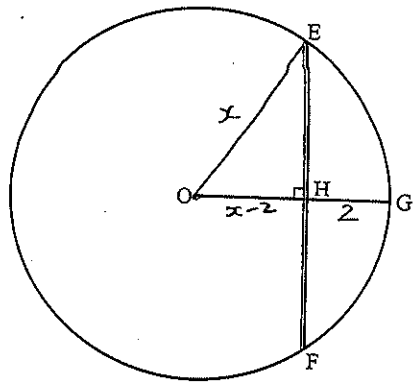
$90° - 2x + \hat{B}_{1+2} + x + x = 180°$ ∠'s Δ = 180°

$\therefore \hat{B}_{1+2} = 90°$ ✓

$\therefore$ AD is diam ✓R conv ∠ in semi ⊙ = 90°

5

9.3.



OH ✓S = x - 2 radii

$EH^2 + (x-2)^2 = x^2$ ✓SR Pythag

$EH^2 + x^2 - 4x + 4 = x^2$

$EH^2 = 4x - 4$

$EH = \sqrt{4x-4}$

$= \sqrt{4(x-1)}$

$= \sqrt{4}\sqrt{x-1}$

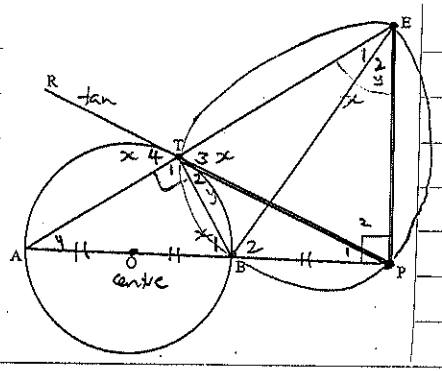
$= 2\sqrt{x-1}$ ✓

$EF = 2 \cdot 2\sqrt{x-1}$

line centre ⊙ ⊥ to chord bisects ✓SR chord

$= 4\sqrt{x-1}$ 4

10.



10.1. $\hat{T}_1 = 90°$ SR ✓ ∠ in semi-⊙ = 90°

but $P_{1+2} = 90°$ given

$\therefore \hat{T}_1 = \hat{P}_{1+2}$ S ✓ both = 90°

$\therefore$ TEPB is cyclic quad ✓R conv ext ∠ = cyclic quad

3

10.2. In Δ's AT_1B_1, $AP_{1+2}E_{1+2}$

1. $\hat{A} = \hat{A}$ ✓ST common

2. $\hat{T}_1 = \hat{P}_{1+2}$ ✓S 1 both = 90°

$\therefore \Delta ATB \;///\; \Delta APE$ AAA ✓R

3

10.3. $\hat{T}_3 \overset{x}{=} \hat{T}_4$ ✓SR vert opp ∠s =

$\hat{T}_4 \overset{x}{=} \hat{B}_1$ ✓SR ∠ in chord

$\hat{B}_1 \overset{x}{=} \hat{E}_{1+2}$ ✓SR Ext ∠ cyc quad

$\therefore \hat{T}_3 \overset{x}{=} \hat{E}_{1+2}$ ✓SR x

$\therefore PT = PE$ ✓S ✓R isos Δ, ∠s =

6

10.4. In Δ's AT_1B_1, $E_2P_{1+2}B_2$

1. $\hat{T}_1 = \hat{P}_{1+2}$ ✓S both = 90°

2. $\hat{E}_2 \overset{y}{=} \hat{T}_2$ ✓SR ∠'s same ⊙ seg =

$\hat{T}_2 \overset{y}{=} \hat{A}$ ✓SR ∠ in chord

$\therefore \hat{E}_2 = \hat{A}$ ✓S y

$\therefore \Delta ATB /// \Delta EPB$ AAA ✓R

5

10.5. $\therefore \Delta A\hat{T}B /// \Delta E\hat{P}B$

$\frac{AB}{EB} = \frac{TB}{PB}$ ✓ // Δ's

$AB.PB = EB.TB$ ✓

but $AB = 2.BP$ given

$\therefore 2BP.PB = EB.TB$ ✓

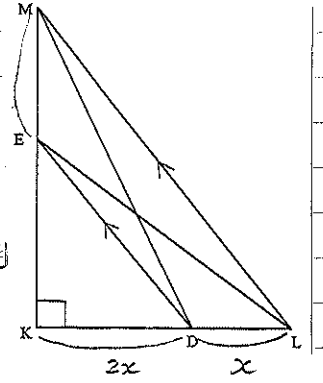
$\therefore 2BP^2 = BE.TB$ ✓

4

11.1. the other 2 sides in the same proportion ✓

1

11.2.



11.2.1. $\frac{KE}{y} = \frac{2x}{x}$ ✓SR line // 1 side Δ

$\therefore KE = 2y$ ✓S

$LM^2 = (3y)^2 + (3x)^2$ ✓SR Pythag

$= 9y^2 + 9x^2$ ✓S

4

11.2.2 RHS $= \frac{13}{9} LM^2$

$= \frac{13}{9}(9y^2 + 9x^2)$

$= 13y^2 + 13x^2$ ✓

LHS $= DM^2 + LE^2$

$= KD^2 + KM^2 + KE^2 + KL^2$ ✓ Pythag

$= (2x)^2 + (3y)^2 + (2y)^2 + (3x)^2$ ✓

$= 4x^2 + 9y^2 + 4y^2 + 9x^2$

$= 13y^2 + 13x^2$ ✓

$\therefore$ LHS = RHS

4